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Massive type-IIA theory on $K3$

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Abstract: In this paper we study $K3$ compactification of ten-dimensional massive type-IIA theory with all possible Ramond-Ramond background fluxes turned on. The resulting six-dimensional theory is a new massive (gauged) supergravity with an action that is manifestly invariant under an $\mathrm{O}(4,20)/\mathrm{O}(4)\times\mathrm{O}(20)$ duality symmetry. We discover that this six-dimensional theory interpolates between vacua of ten-dimensional massive IIA supergravity and vacua of massless IIA supergravity with appropriate background fluxes turned on. This in turn suggests a new 11-dimensional interpretation for the massive type-IIA theory.

Keywords: M-Theory, String Duality, Supersymmetry and Duality, Supergravity Models.

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Contents

1. Introduction

Recently there has been renewed interest in the study of gauged/massive supergravity theories due to the AdS/CFT correspondence [1]. In gauged supergravity theories either a subgroup of the R-symmetry group (the automorphism group of the supersymmetry algebra) or isometries of the scalar manifold are gauged by some of the vector fields in the spectrum [2]. Such theories can be constructed from their ungauged "cousins" by adding appropriate terms to the action or field equations and changing the supersymmetry transformation laws accordingly. This procedure does not change the spectrum and the number of supercharges but generically does change the properties of the ground state. For example, a Minkowskian spacetime which always is a solution of the ungauged theory ceases to be a ground state of the gauged supergravity in most cases. Instead the supersymmetric ground states are often of the anti-de-Sitter (AdS) type or domain-wall solutions.

A particular subclass of gauged supergravities are the massive supergravities. In these theories some of the vector (or tensor) fields become massive through a generalized Higgs mechanism. Such theories occur for a specific gauging which allows the possibility of a Higgs-type mechanism. A prominent example is the massive type-IIA supergravity in $D = 10$ constructed by Romans [3]. In string theory massive supergravities typically arise in lower dimensions through generalized Scherk-Schwarz reduction [4] provided that some field strength F_{p+1} of the p-form tensor field A_p is given a non-trivial background value (flux) along the compact directions [5, 6, 7]. Such background fluxes can be turned on consistently if the action or the field equations depend on A_p only through its field strength F_{p+1}.

Gauged supergravities have also been studied in the context of string dualities and D-branes. For example, the massive type-IIA supergravity has a domain wall solution which preserves 1/2 of the 32 supercharges of type-IIA [5] and can be given an interpretation of a type-IIA D-8-brane [8]. This observation has led to the search for possible duality connections involving massive supergravity theories analogous to the existing U-duality relations in the massless cases. This required the construction of other massive supergravities in lower dimensions through generalized dimensional reduction [5, 6, 7], [9]–[17]. However, it has remained an interesting and open question to what extent these generalized compactifications respect the duality properties of the massless cases. Along this line, Kaloper and Myers [16] showed that a generalized Scherk-Schwarz reduction of the heterotic string on a d-torus can still be written in a manifestly $\mathrm{O}(d, d+16)$ invariant form provided T-duality transformations also act on the background fluxes.

In a parallel line of developments Calabi-Yau compactifications with background fluxes have also been studied because of their phenomenological properties [18]–[38]. One finds that background fluxes generically generate a potential for some of the moduli fields of the theory without fluxes and as a consequence the moduli space — and hence the arbitrariness of the theory — is reduced. In addition the resulting ground states can break supersymmetry spontaneously.

In this paper we study a generalized $K3$ reduction of ten-dimensional massive type-IIA theory with all possible background fluxes turned on. Our goal is to investigate the fate of the perturbative $\mathrm{O}(4,20)$ duality symmetry and the non-perturbative S-duality with the heterotic string compactified on $\mathbb{T}^4$. We also discuss the relation of massive type-IIA theory with M-theory. The paper is organized as follows. In section 2 we recall the massive type-II theory and derive its $K3$ compactification with all R-R background fluxes turned on. We find that the resulting six-dimensional theory is a new massive (gauged) supergravity with manifest $\mathrm{O}(4,20)$ symmetry. In section 3 we study domain wall solutions of this massive supergravity. We show that the D-8-brane of massive type IIA when wrapped on $K3$ is related via $\mathrm{O}(4,20)$ duality to a solution of ordinary type IIA on $K3$ with an appropriate four-form flux turned on. Thus perturbative T-duality symmetry interpolates between vacua of massive and massless type-IIA theories on $K3$. This property also allows us to further relate massive type-II vacua to eleven dimensional solutions. This is discussed in section 4 where massive IIA theory is given a 11-dimensional interpretation. Finally we conclude in the section 5.

2. $K3$ compactification of massive type-IIA supergravity

The type-IIA supergravity in ten dimensions, which describes the low energy limit of type-IIA superstrings, contains in the massless spectrum the graviton $\hat{g}_{MN}$, the dilaton $\hat{\phi}$, an NS-NS two-form $\hat{B}_2$, a R-R one-form $\hat{A}_1$ and a R-R three-form $\hat{C}_3$. The fermionic fields consist of two gravitini and two Majorana spinors. It was shown by Romans [3] that this supergravity can be generalized to include a mass term for the $\hat{B}$-field without disturbing the supersymmetry. The $\hat{B}$-field becomes massive through a Higgs type mechanism in which it eats the vector field $\hat{A}_1$. The supersymmetric action for massive IIA theory in the string frame is given by [3]

$$\begin{aligned} S = \int \Big[& e^{-2\hat{\phi}} \left(\frac{1}{4} \hat{R} \, \hat{*}1 + d\hat{\phi} \, \hat{*} d\hat{\phi} - \frac{1}{2} \hat{H}_3 \, \hat{*} \hat{H}_3 \right) - \frac{1}{2} \hat{F}_2 \, \hat{*} \hat{F}_2 - \frac{1}{2} \hat{F}_4 \, \hat{*} \hat{F}_4 - \frac{m^2}{2} \, \hat{*}1 + \\ & + d\hat{C}_3 d\hat{C}_3 \hat{B}_2 + 2 d\hat{C}_3 d\hat{A}_1 \hat{B}_2^2 + \frac{4}{3} d\hat{A}_1 d\hat{A}_1 \hat{B}_2^3 + \frac{4}{3} m d\hat{C}_3 \hat{B}_2^3 + 2 m d\hat{A}_1 \hat{B}_2^4 + \\ & + \frac{4}{5} m^2 \hat{B}_2^5 \Big] + \text{fermionic part} \,, \end{aligned} \tag{2.1}$$

where we have adopted the notation that every product of forms is understood as a wedge product. The signature of the metric is $(- + \cdots +)$ and for a p-form we use the convention

$$F_p = \frac{1}{p!} F_{M_1 \cdots M_p} dx^{M_1} \wedge \cdots \wedge dx^{M_p} \,, \tag{2.2}$$

while the Poincare dual is given by

$$\begin{aligned} \hat{*} F_p &= \frac{1}{p!(10-p)!} F_{M_1 \cdots M_p} \epsilon^{M_1 \cdots M_p}{}_{M_{p+1} \cdots M_{10}} \, dx^{M_{p+1}} \wedge \cdots \wedge dx^{M_{10}} \,, \\ \hat{*}\hat{*} F_p &= -(-1)^{p(10-p)} F_p \,, \end{aligned} \tag{2.3}$$

and $\hat{*}1 = \sqrt{-\hat{g}} \, d^{10}x$. m is the mass parameter and the various field strengths in the lagrangian (2.1) are defined as

$$\hat{H}_3 = d\hat{B}_2 \,, \qquad \hat{F}_2 = d\hat{A}_1 + 2m\hat{B}_2 \,, \qquad \hat{F}_4 = d\hat{C}_3 + 2\hat{B}_2 d\hat{A}_1 + 2m\hat{B}_2^2 \,. \tag{2.4}$$

$\hat{A}_1$ and $\hat{C}_3$ only appear through their derivatives in the lagrangian (2.1) and thus obey the standard p-form gauge invariance $A_p \to A_p + d\Lambda_{p-1}$. The two-form $\hat{B}_2$ on the other hand also appears without derivatives but nevertheless the "Stueckelberg" gauge transformation

$$\delta \hat{A}_1 = -2m\Lambda_1 \,, \qquad \delta \hat{B}_2 = d\Lambda_1 \,, \qquad \delta \hat{C}_3 = -2\Lambda_1 d\hat{A}_1 \tag{2.5}$$

leave the lagrangian invariant. Finally, the conventional massless type-IIA theory is recovered from the action (2.1) in the limit $m \to 0$.

Before we turn to the compactification let us first recall some facts about $K3$ manifolds. $K3$ is a compact, Ricci flat complex manifold with Betti numbers $b_0 = 1$, $b_1 = 0$, $b_2 = 22$, $b_3 = 0$, $b_4 = 1$. Thus there exist 22 harmonic two-forms Ω_2^i $(i = 1, \ldots, 22)$ and their intersection matrix

$$\eta_{ij} = \int_{K3} \Omega_2^i \wedge \Omega_2^j \tag{2.6}$$

is a Lorentzian metric with signature $(3,19)$. We choose conventions for the two-forms Ω_2^i such that η is given by

$$\eta_{ij} = \begin{pmatrix} 0 & 0 & \sigma \\ 0 & I_{16} & 0 \\ \sigma & 0 & 0 \end{pmatrix}, \qquad \text{where} \qquad \sigma = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{2.7}$$

and I_n represents the n-dimensional identity matrix. Since $K3$ is four-dimensional the Hodge-dual of the harmonic 2-forms can again be expanded in terms of two-forms. More precisely one has

$$^*\Omega_2^i = M^i{}_j\ \Omega_2^j\,, \tag{2.8}$$

where the matrix $M^i{}_j$ depends on the $3 \times 19 = 57$ moduli parameterizing the deformations of the metric δg_{mn} of constant volume on $K3$. For $K3$ $^{**} = 1$ holds which implies

$$M^i{}_j M^j{}_k = \delta^i_k\,, \qquad \eta_{ij} M^j{}_k = \eta_{kj} M^j{}_i\,, \tag{2.9}$$

so that $M^j{}_i \eta_{jk} M^k_l = \eta_{il}$. Thus M is an element of the coset $\mathrm{O}(3,19)/\,\mathrm{O}(3)\times\mathrm{O}(19)$ [39, 40].

Let us now turn to the compactification of the massive type-IIA theory on $K3$. The standard Kaluza-Klein reduction considers the theory in a spacetime background $M_D \times K_d$, where M_D is a non-compact D-dimensional manifold with Lorentzian signature while K_d is a d-dimensional compact manifold. This ansatz is consistent whenever the spacetime background satisfies the $D+d$-dimensional field equations. However, for massive type IIA there are no direct product $M_6 \times K3$ solutions; instead the ground states are domain-wall solutions [5]. A similar situation occurs in the S^1 compactification of massive type IIA discussed in [5]. However, as argued there one can also expand around a solution which strictly speaking is not a direct product $M_D \times K_d$, but rather the compact manifold K_d is allowed to vary over the spacetime manifold M_D.[1] For the case at hand such a solution exists and is given by the D-8-brane solution of massive type-IIA theory wrapped on a $K3$ [5, 42]. We discuss this solution in the next section and will find that it can be interpreted as the product of a 6-dimensional domain wall with a warped $K3$ whose volume varies over the transverse direction. This spacetime dependence of the volume ensures that the equations of motion of massive type-IIA theory are fulfilled.

Thus for the 10-dimensional metric we take the standard ansatz

$$\hat{G}_{MN}(x,z) = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & g^0_{mn}(z) + \delta g_{mn}(x,z) \end{pmatrix}, \tag{2.10}$$

where $g_{\mu\nu}(x)$ is the metric and x are the coordinates of M_6. $g^0_{mn}(z)$ is a fixed back-

[1] Or in other words the moduli of K_d are not constant in the background but vary over M_D.

ground metric on $K3$ (with coordinates z) and $\delta g_{mn}(x,z)$ denotes the allowed deformations of the $K3$ metric. These deformations are parameterized by $57+1$ moduli where the extra modulus corresponds to the overall volume of $K3$.[2]

For the dilaton and the two-form $\hat{B}_2$ we take the standard ansatz precisely as in massless type-IIA theories

$$\hat{\phi}(x,z) = \hat{\phi}(x)\,, \qquad \hat{B}_2(x,z) = B_2(x) + b^i(x)\ \Omega_2^i(z)\,, \tag{2.11}$$

where $B_2(x)$ is a two-form in $D=6$ and the $b^i(x)$ are 22 additional scalar fields. (Thus the total number of scalars is $58+22+1=81$.)

For the one-form $\hat{A}_1$ and the three-form $\hat{C}_3$ we take a generalized Kaluza-Klein ansatz where background values (fluxes) of the corresponding field strengths are included

$$\begin{aligned} d\hat{A}_1(x,z) &= dA_1(x) + m^i\ \Omega_2^i(z)\,, \\ d\hat{C}_3(x,z) &= dC_3(x) + dC_1^i(x)\ \Omega_2^i(z) - \tilde{m}\ \Omega_4(z)\,. \end{aligned} \tag{2.12}$$

This ansatz introduces 23 new mass parameters m^i and $\tilde{m}$ parameterizing fluxes along the 22 two-cycles and the four-cycle on $K3$. This generalization is possible since $\hat{A}_1, \hat{C}_3$ appear in the action only with derivative couplings (i.e. via their field strength) and an appropriate background value can be consistently turned on [7].

Altogether, the bosonic spectrum of the reduced six-dimensional theory consists of the graviton $g_{\mu\nu}$, the two form B_2, 22+1 one-form gauge fields (C_1^i, A_1), a three-form C_3 and 81 scalar fields $\phi, b^i, \delta g_{mn}$. From their definition it is clear that C_1^i and b^i both transform in the vector representation of $\mathrm{O}(3,19)$. In $D=6$ a three-form is Poincare dual to a one-form and thus the above spectrum combines into a gravitational multiplet consisting of the graviton, the two-form, four vector fields and the dilaton and 20 vector multiplets each containing a one-form and four scalars. This is the bosonic content of $N=(1,1)$ supergravity theory in $D=6$ coupled to twenty vector multiplets.

In order to obtain the action of the massless modes for this theory we substitute the ansatz (2.10)–(2.12) into the action (2.1). The resulting six-dimensional bosonic action reads

$$\begin{aligned} S_6 = \int \Big[&\frac{1}{4} e^{-2\phi} \Big(R\ {}^*1 + 4 d\phi\ {}^*d\phi - 2 H_3\ {}^*H_3 + \frac{1}{8} \operatorname{Tr} d\mathcal{M}^{-1}\ {}^*d\mathcal{M} \Big) - \\ &- \frac{1}{2} \mathcal{F}_2^a \left(\mathcal{M}^{-1}\right)_{ab}\ {}^*\mathcal{F}_2^b - \frac{1}{2} m^a \left(\mathcal{M}^{-1}\right)_{ab}\ {}^*m^b + B_2 \mathcal{F}_2^a \mathcal{L}_{ab} \mathcal{F}_2^b - \\ &- 2 B_2^2 m^a \mathcal{L}_{ab} \mathcal{F}_2^b + \frac{4}{3} B_2^3 m^a \mathcal{L}_{ab} m^b \Big]\,, \end{aligned} \tag{2.13}$$

[2] The metric on the moduli space of sigma models on $K3$ does not change with respect to the standard Kaluza-Klein reduction of massless type-IIA theory and can therefore be taken from that case [39, 40, 41].

where

$$2\phi = 2\hat{\phi} - \ln\omega\,, \qquad H_3 = dB_2\,, \qquad \mathcal{F}_2^a = d\mathcal{A}_1^a + 2m^a B_2\,. \tag{2.14}$$

The index a takes values $a = 1, \ldots, 24$ and we have defined

$$m^a \equiv (\tilde{m},\ m^i,\ m)\,, \qquad \mathcal{A}_1^a \equiv (\tilde{A}_1,\ C_1^i,\ A_1)\,. \tag{2.15}$$

$d\tilde{A}_1$ is the Poincare dual of the 4-form dC_3 defined as

$$\begin{aligned} d\tilde{A}_1 = & -\omega\ {}^*\left(dC_3 + 2B_2 dA_1 + 2mB_2^2\right) + 2b_i\left(dC_1^i + b^i dA_1\right) + \\ & + 2\left(-\tilde{m} + 2b_i(m^i + m\ b^i)\right) B_2\,, \end{aligned} \tag{2.16}$$

where the indices i, j are contracted with the metric η_{ij} and $\omega(x)$ is the modulus associated with the overall volume V of the $K3$

$$V = \omega(x) \int_{K3} \Omega_4\,. \tag{2.17}$$

From eq. (2.16) we learn that $d\tilde{A}$ is an $\mathrm{O}(3,19)$ invariant 2-form field strength. Finally, the scalar matrix $\mathcal{M}^{ab}$ which appears in the action (2.13) depends on the 58 moduli of $K3$ and the 22 b^i in the following way

$$\mathcal{M}^{-1} = \mathcal{V}^T\mathcal{V}\,, \qquad \mathcal{V} = \begin{pmatrix} \omega^{-1/2} & -2\omega^{-\frac{1}{2}}\bar{b} & -2\omega^{-\frac{1}{2}}\bar{b}b \\ 0 & v & 2vb \\ 0 & 0 & \omega^{1/2} \end{pmatrix}, \tag{2.18}$$

where $\bar{b} \equiv (\eta\ b)^T = b^T\eta$. The 22×22 sub-matrix v depends only on the 57 moduli of $K3$ (without the volume) and determines the inverse of the matrix $M^{ij} = M^i{}_k\eta^{kj}$ introduced in (2.8)

$$M^{-1} = v^T v\,, \qquad v\eta v^T = \eta\,. \tag{2.19}$$

The matrices $\mathcal{M}$ and $\mathcal{V}$ satisfy

$$\mathcal{V}\mathcal{L}\mathcal{V}^T = \mathcal{L}\,, \qquad \mathcal{M}\mathcal{L}\mathcal{M}^T = \mathcal{L}\,, \qquad \mathcal{M}^T = \mathcal{M}\,, \tag{2.20}$$

where $\mathcal{L}$ is the $\mathrm{O}(4,20)$ metric

$$\mathcal{L} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \eta & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{2.21}$$

The action (2.13) is invariant under global $\mathrm{O}(4,20)$ transformations acting according to

$$\begin{aligned} &\mathcal{M} \longrightarrow U\mathcal{M}\,U^T\,, \qquad \mathcal{A}^a \longrightarrow U^a{}_b\,\mathcal{A}^b\,, \qquad m^a \longrightarrow U^a{}_b\,m^b\,, \\ &\phi \longrightarrow \phi\,, \qquad B \longrightarrow B\,, \qquad g_{\mu\nu} \longrightarrow g_{\mu\nu}\,, \end{aligned} \tag{2.22}$$

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where $U \in \mathrm{O}(4,20)$. These symmetry transformations except the transformations of the mass parameters are precisely the perturbative duality transformations of the standard (massless) type-IIA theory on $K3$. Indeed, in the limit $m^a \to 0$ the action (2.13) reduces to the action of massless type IIA on $K3$ [41]. In the massive case the symmetry can be maintained if the 24 mass parameters m^a transform in the vector representation of $\mathrm{O}(4,20)$. This transformation of masses means that under the action of the duality group a massive IIA compactified on $K3$ with fluxes transforms into another massive IIA with a different set of background fluxes as determined by the symmetry. We emphasize that the action (2.13) represents a unification of a wide class of massive supergravities related to each other via the action of $\mathrm{O}(4,20)$ symmetry. Any particular choice of the mass vector m^a represents a different massive theory. For example, if we set $m^a = (0,\ldots,0,m)$ in the action (2.13) the theory represents a pure massive IIA compactified on $K3$ without any fluxes. Similarly a different choice $m^a = (\tilde{m},0,\ldots,0)$ in the above action represents a six-dimensional massive theory obtained through generalized reduction of type IIA on $K3$ with 4-form flux. We will show explicitely in the next sections that these two theories with single mass parameters are related via an element of the duality symmetry (2.22).

Apart from ordinary gauge invariance $\mathcal{A}_1^a \to \mathcal{A}_1^a + d\Lambda_0^a$ the field strengths given in (2.14), the Bianchi identities $d\mathcal{F}_2^a = 2m^a\, H_3,\ dH_3 = 0$ as well as the action (2.13) exhibit a Stueckelberg type gauge symmetry of the form

$$\delta B_2 = d\Lambda_1\,, \qquad \delta\mathcal{A}_1^a = -2m^a\Lambda_1\,, \tag{2.23}$$

where Λ_1 is a one-form. This gauge invariance can be used to absorb one of the 24 gauge fields into B_2 and render it massive. Such a gauge fixed version of the theory breaks the $\mathrm{O}(4,20)$ symmetry spontaneously since in a given vacuum only one of the A-fields can be absorbed. Nevertheless, at the level of the action the $\mathrm{O}(4,20)$ symmetry is manifest and will play an important role in determining new vacuum configurations.

3. Domain wall solutions

Generally, massive supergravities admit domain-wall solutions which preserve half of the supersymmetries. So we also expect this to be the case for the six-dimensional massive theory in (2.13). It is known that the ten-dimensional massive IIA theory has a D-8-brane (domain-wall) solution which preserves 16 supercharges [5]. In the string frame metric it is given by

$$\begin{aligned} d\hat{s}^2 &= H^{-1/2}\left(-dt^2 + \sum_{i=1}^{8} dx_i^2\right) + H^{1/2}dy^2\,, \\ 2\hat{\phi} &= -\frac{5}{2}\ln H\,, \end{aligned} \tag{3.1}$$

where $H = 1 + 2m|y - y_0|$ is a harmonic function of the transverse coordinate y and all other fields have vanishing backgrounds. This solution can be compactified by wrapping four of the world-volume directions on $K3$. In other words one can also write a D-8-brane solution with four of its brane directions being along $K3$

$$\begin{aligned} d\hat{s}^2 &= H^{-1/2}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2 + ds_{K3}^2\right) + H^{1/2}dy^2\,, \\ 2\hat{\phi} &= -\frac{5}{2}\ln H\,, \qquad H = 1 + 2m|y - y_0|\,. \end{aligned} \tag{3.2}$$

That this is indeed a solution of the equations of motion has been shown in [42], where it is argued that one can replace the spatial part of the D-8-brane's worldvolume by any manifold of the form $\mathbb{R}_{8-d} \times K_d$ with a Ricci-flat K_d. It is further shown that for $K_d = K3$ the solution preserves 8 supercharges.[3] The corresponding six-dimensional domain-wall solution can be written as

$$\begin{aligned} ds_6^2 &= H^{-1/2}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2\right) + H^{1/2}dy^2\,, \\ 2\phi &= 2\hat{\phi} - \ln\omega = -\frac{3}{2}\ln H\,, \\ m^a &= (0, \dots, 0,\ m)\,, \\ \mathcal{M}^{-1} &= \mathrm{diag}\left(H, 1, \dots, 1, H^{-1}\right)\,, \end{aligned} \tag{3.3}$$

while all other six-dimensional background values vanish. The breathing mode ω defined in (2.17) is given by H^{-1} in this example. Furthermore in (3.3) we have chosen a special point in the moduli space of $K3$ where (2.8) reads

$$\begin{aligned} &{}^*\Omega_2^i = \Omega_2^{23-i}\,, \qquad {}^*\Omega_2^{23-i} = \Omega_2^i \qquad \text{for} \qquad i = 1, 2, 3\,; \\ &{}^*\Omega_2^j = \Omega_2^j\,, \qquad \text{for} \qquad j = 4, \dots, 19\,. \end{aligned} \tag{3.4}$$

The solution (3.3) still has 8 unbroken supersymmetries.

Now, by applying an $O(4, 20)$ duality transformation (2.22) on the background in (3.3) new solutions with non-trivial R-R fluxes can be generated. Let us first consider the special case where the matrix U is taken to be

$$U = \begin{pmatrix} 0 & 0 & 1 \\ 0 & I_{22} & 0 \\ 1 & 0 & 0 \end{pmatrix}\,. \tag{3.5}$$

[3] As anticipated in section 2 the solution (3.2) is a warped product of a six-dimensional domain wall and a $K3$.

Inserting U and the configuration (3.3) in (2.22) we get $\omega \to \omega' = 1/\omega$ and

$$m^a = \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ m \end{pmatrix} \longrightarrow m'^a = \begin{pmatrix} m \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix},$$
$$\mathcal{M}^{-1} = \begin{pmatrix} H & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & H^{-1} \end{pmatrix} \longrightarrow \mathcal{M}'^{-1} = \begin{pmatrix} H^{-1} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & H \end{pmatrix}, \tag{3.6}$$

while the six-dimensional metric and the dilaton remain the same. The transformed mass vector m'^a implies that the new configuration is a solution of a massless IIA compactified on $K3$ with an equivalent amount of four-form flux turned on along $K3$. When (3.6) is lifted to ten dimensions we get the following new configuration

$$d\hat{s}'^2 = H^{-1/2}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2\right) + H^{1/2}\left(dy^2 + ds^2_{K3}\right),$$
$$2\hat{\phi}' = -\frac{1}{2}\ln H\,, \qquad \hat{F}'_4 = -m\Omega_4\,. \tag{3.7}$$

Since this solution is obtained by the duality transformation (2.22) the number of preserved supercharges is unchanged. It can also be checked explicitly that (3.7) leaves 8 supercharges unbroken. The background (3.7) represents a stack of D-4-branes filling the $K3$ with non-trivial 4-form flux. This configuration can be further lifted to eleven dimensions as we will see in the next section.

By applying an O$(4,20)$ duality transformation we transformed a solution of massive type IIA to a solution of massless type IIA with non-trivial four-form flux. Thus, the O$(4,20)$ duality interpolates between vacua of massive IIA and massless IIA. In spirit this is similar to the situation encountered in the case of massive type-II duality in $D = 9$ [5].

Further solutions in $D = 6, 10$ can be generated by using other elements of the duality group which mix the mass m with the fluxes m^i of the 2-cycles. Let us consider the case

$$U = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{20} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \tag{3.8}$$

which mixes m and m_{22}. Note that the duality element in (3.8) preserves the O$(4,20)$ metric as can be checked using the explicit form for η given in (2.7). When we apply (3.8) on the configuration (3.3), using (2.22) we find $\omega \to 1$ and

$$m^a = \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ 0 \\ m \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ m \\ 0 \end{pmatrix} ,$$

$$\mathcal{M}^{-1} = \begin{pmatrix} H & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & H^{-1} \end{pmatrix} \longrightarrow \mathcal{M}'^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & H & 0 & 0 & 0 \\ 0 & 0 & I_{20} & 0 & 0 \\ 0 & 0 & 0 & H^{-1} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} , \tag{3.9}$$

while all other six-dimensional fields remaining unchanged. Again from the analysis of the new mass-vector it can be seen that this new six-dimensional configuration corresponds to massless IIA compactified on $K3$ but now with a two-form flux along a 2-cycle of $K3$. When lifted to ten dimensions we have a new solution of massless IIA theory as

$$\begin{aligned} d\hat{s}'^2 &= H^{-1/2}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2\right) + ds'^2_{K3} + H^{1/2}dy^2 \,, \\ 2\hat{\phi}' &= -\frac{3}{2}\ln H \,, \qquad \hat{F}'_2 = m\Omega_2^{(22)} \,, \end{aligned} \tag{3.10}$$

where ds'^2_{K3} is the deformed metric on $K3$ such that the deformation corresponds to the non-trivial moduli matrix $\mathcal{M}'^{-1}$ in (3.9). $\Omega_2^{(22)}$ is a 2-form on $K3$. This background configuration (3.10) preserves the same amount of supersymmetries as the one in (3.3).

One can go through a similar analysis for solutions which depend on more than one mass or flux parameter. Let us consider the following solution of the massive IIA theory in (2.1)

$$\begin{aligned} d\hat{s}^2 &= H^{-1/2}H'^{-1/2}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2\right) + H^{1/2}H'^{-1/2}ds^2_{K3} + H^{1/2}H'^{1/2}dy^2 \,, \\ 2\hat{\phi} &= -\frac{1}{2}\ln(HH'^5) \,, \qquad \hat{F}_4 = -\tilde{m}\,\Omega_4 \,, \qquad \hat{F}_2 = 0 = \hat{B}_2 \,, \\ H' &= 1 + 2m|y| \,, \qquad H = 1 + 2\tilde{m}|y| \,. \end{aligned} \tag{3.11}$$

For the special case when $\tilde{m} = m$, $H' = H$ it reduces to

$$\begin{aligned} d\hat{s}^2 &= H^{-1}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2\right) + H\ dy^2 + ds^2_{K3} \,, \\ 2\hat{\phi} &= -3\ln H \,, \qquad \hat{F}_4 = -\tilde{m}\,\Omega_4 \,, \qquad \hat{F}_2 = 0 = \hat{B}_2 \,, \end{aligned} \tag{3.12}$$

which preserves 8 supercharges as can be checked explicitly. This can be compactified to six dimensions and using the U-matrix of (3.8) it can be transformed as

$$m^a = \begin{pmatrix} m \\ 0 \\ \cdot \\ \cdot \\ 0 \\ m \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ m_1 = m \\ 0 \\ \cdot \\ \cdot \\ 0 \\ m_{22} = m \\ 0 \end{pmatrix},$$
$$\mathcal{M}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.13}$$

Lifting this solution back to ten dimensions we get

$$d\hat{s}^2 = H^{-1}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2\right) + H\ dy^2 + ds^2_{K3}\,,$$
$$2\hat{\phi} = -3\ln H\,, \qquad \hat{F}_4 = 0\,, \qquad \hat{F}_2 = m_1\Omega_2^{(1)} + m_{22}\Omega_2^{(22)}\,. \tag{3.14}$$

$\hat{F}_2$ is self-dual since $m_1 = m_{22} = m$. This is the solution obtained in [7] for massless type IIA.

4. A lift to M-theory

It is conjectured that the strong coupling limit of type IIA is governed by M-theory [43] whose low energy limit is believed to be 11-dimensional supergravity [44]. In fact one can obtain massless type-IIA supergravity as an S^1 compactification of 11-dimensional supergravity and in this way all p-brane solutions of massless type IIA can be lifted to eleven dimensions. However, the 8-brane solution of massive IIA given in (3.1) cannot be lifted easily to 11-dimensions since that would require a massive version of 11-dimensional supergravity [11, 13] which does not exist [45, 46]. In ref. [15] a general relation between massive IIA theory and M- and F-theory has been proposed. We will see here that taking the detour of compactifying massive IIA on $K3$ provides another possibility to relate the 8-brane solution to M-theory.[4] In order to show this in slightly more detail let us write down the map among the fields of massive and massless type IIA on $K3$. As discussed above, the six-dimensional theory in (2.13) with the mass vector $m^a = (\tilde{m}, 0, \ldots, 0)$ represents an ordinary type IIA on $K3$ with RR-4-flux. On the other hand the theory with mass vector

[4] It would be interesting to understand the connection with ref. [15] in more detail.

$m^a = (0, \ldots, 0, m)$ represents massive IIA on $K3$ without any RR-flux. These two massive six-dimensional theories are related by the duality element (3.5). In section 3, eqs. (3.3)–(3.7) we displayed the duality relation between a domain wall solution of massive IIA and a stack of solitonic D-4-branes of massless type IIA. In fact one can not only map the solutions onto each other but the entire actions. Under the transformation given in (3.5) massive IIA with no fluxes transforms into massless IIA with 4-form flux as

$$\begin{aligned}
\omega^{-1} &\longrightarrow \omega + 4b^T M^{-1} b + \frac{4(b^2)^2}{\omega}\,, \\
-\frac{2\bar{b}}{\omega} &\longrightarrow \frac{4b^2\bar{b}}{\omega} + 2b^T M^{-1}\,, \qquad M^{-1} \longrightarrow M^{-1}\,, \qquad \frac{2b^2}{\omega} \longrightarrow \frac{2b^2}{\omega}\,, \\
d\tilde{A} &\longrightarrow dA + 2mB\,, \qquad dA + 2mB \longrightarrow d\tilde{A}\,, \qquad dC^i \longrightarrow dC^i\,, \\
\phi &\longrightarrow \phi\,, \qquad B \longrightarrow B\,, \qquad g_{\mu\nu} \longrightarrow g_{\mu\nu}\,.
\end{aligned} \tag{4.1}$$

Under this map the mass parameter m of massive type IIA is mapped to the four-form flux $\tilde{m}$ on $K3$ of massless IIA and vice versa.

Since massless type IIA on $K3$ is equivalent to M-theory on $S^1 \times K3$, all solutions of massive IIA can be lifted to eleven dimensions by first mapping them to solutions of the massless theory using the map (4.1). The solution given in (3.7) corresponds to the following eleven dimensional solution

$$\begin{aligned}
ds^2_{(11)} &= e^{4\hat{\phi}/3}\left(dx_{11} + \hat{A}^M dx_M\right)^2 + e^{-2\hat{\phi}/3} ds^2_{(10)} \\
&= H^{-1/3}\left(-dt^2 + \sum_{i=1}^{4} dx_i^2 + dx_{11}^2\right) + H^{2/3}\left(dy^2 + ds^2_{K3}\right), \\
G_4 &= -m\ \Omega_4\,, \qquad H = 1 + 2m|y|\,.
\end{aligned} \tag{4.2}$$

where G_4 is the 4-form field strength of eleven-dimensional supergravity. The solution (4.2) is a stack of M5-branes which couple magnetically to the 4-form field strength.[5]

Thus it seems that the conjectured duality between type IIA compactified on $K3$ and M-theory on $K3 \times S^1$ extends to the massive case. It is shown in ref. [10] that for M-theory on $K3 \times S^1$ the compactifications on $K3$ and S^1 commute even in the presence of 4-form flux on $K3$. Here we have seen that massless type IIA compactified on $K3$ with "4-form flux along $K3$" is T-dual to massive IIA compactified on $K3$ without flux. Thus we are led to conjecture that M-theory on $K3 \times S^1$ with "4-form flux on $K3$" is dual to massive type IIA compactified on a $K3$ without flux. In

[5] It is possible to establish a similar map as in (4.1) between the backgrounds of massive IIA compactified on $K3$ (with $m^i \neq 0$) and the backgrounds of massless IIA on $K3$ (with $m^i \neq 0$). However, it is not clear whether those can be lifted to 11 dimensions since this would require a globally defined $\hat{A}_1$.

this duality the mass of Romans theory is mapped to the 4-form flux of M-theory along $K3$.

$$\begin{array}{ccc} & & \boxed{\text{M-theory on } S^1 \times K3 \text{ \& 4-Flux}} \\ & & \updownarrow \\ \boxed{\text{massive IIA on } K3} & \xleftrightarrow{(\omega \leftrightarrow 1/\omega)} & \boxed{\text{massless IIA on } K3 \text{ \& 4-Flux}} \end{array}$$

5. Conclusion

In this paper we derived the action for $K3$ compactifications of ten-dimensional massive type-IIA theory with all Ramond-Ramond background fluxes turned on. We found that the resulting six-dimensional theory is a new massive (gauged) supergravity with an action having manifest $\frac{\mathrm{O}(4,20)}{\mathrm{O}(4)\times\mathrm{O}(20)}$ duality symmetry provided the mass (flux) parameters transform accordingly. From this we learn that the perturbative T-duality survives even at the massive level when appropriate masses and fluxes are switched on.

We have seen in this paper that the massive six-dimensional theory of (2.13) interpolates between ten-dimensional massive and massless type-IIA theories. The wrapped D-8-brane solution of massive type-IIA turns out to be T-dual to a supersymmetric solution of massless IIA theory on $K3$ with four-form flux. The relationship between massless and massive IIA on $K3$ also suggests a new 11-dimensional interpretation of massive IIA theory.

As it is known, in addition to the perturbative T-duality there also is a non-perturbative S-duality between massless type IIA on $K3$ and massless heterotic string on $\mathbb{T}^4$ [39, 40, 41]. Both theories have the same perturbative T-duality group $\frac{\mathrm{O}(4,20)}{\mathrm{O}(4)\times\mathrm{O}(20)}$ and their respective dilatons are related by $\phi_{\mathrm{Het}} \to -\phi_{\mathrm{IIA}}$. Let us recall that there also exists an $\mathrm{O}(d, d+n)$ symmetric massive compactification of the heterotic string on $\mathbb{T}^d$ [16]. However, the non-perturbative S-duality seems no longer to be valid in the massive theories. The major difference is that in the type-IIA theory (2.13) it is the tensor field B which becomes massive after eating the vector fields while in the heterotic theory vector fields become massive after eating some scalars [16]. This is similar to the situation encountered in the duality between massive M-theory on $K3$ and heterotic theory on $\mathbb{T}^3$ [7]. Furthermore there is a runaway dilaton potential in both theories driving the dilaton to weak coupling. As a consequence strong-weak S-duality can no longer hold.

Finally, it is interesting to consider a further reduction of the massive six-dimensional theory (2.13) on S^1 and explore some duality relationship with type IIB on $K3 \times S^1$ with fluxes. It is clear from the action (2.13) that there are no axions in this theory so the further reduction on S^1 will not produce any new mass parameter. On the other hand the standard reduction of type IIB on $K3$ produces exactly 24 axions in six dimensions which upon further generalized compactification on S^1 will generate 24 mass parameters in five dimensions. Through a duality relation one should be able to relate the 24 mass parameters of massive IIA on $K3 \times S^1$ to those of type IIB on $K3 \times S^1$. This should be related to the duality of massive type IIA and type IIB in $D = 9$ [5].

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References

[1] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200];
for a review see O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rep.* **323** (2000) 183 [hep-th/9905111].

[2] For a collection of reprints see, *Supergravities in diverse dimensions*, A. Salam and E. Sezgin eds., World Scientific, Singapore 1989.

[3] L.J. Romans, *Massive $N = 2a$ supergravity in ten-dimensions*, *Phys. Lett.* **B 169** (1986) 374.

[4] J. Scherk and J.H. Schwarz, *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61.

[5] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Duality of type II 7-branes and 8-branes*, *Nucl. Phys.* **B 470** (1996) 113 [hep-th/9601150].

[6] P.M. Cowdall, H. Lü, C.N. Pope, K.S. Stelle and P.K. Townsend, *Domain walls in massive supergravities*, *Nucl. Phys.* **B 486** (1997) 49 [hep-th/9608173].

[7] I.V. Lavrinenko, H. Lü and C.N. Pope, *From topology to generalised dimensional reduction*, *Nucl. Phys.* **B 492** (1997) 278 [hep-th/9611134].

[8] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017].

[9] T. Inagaki, S. Mukaigawa and T. Muta, *A soluble model of four fermion interactions in de Sitter space*, *Phys. Rev.* **D 52** (1995) 4267 [hep-th/9505058].

[10] H. Lü and C.N. Pope, *Domain walls from M-branes*, *Mod. Phys. Lett.* **A 12** (1997) 1087 [hep-th/9611079].

[11] E. Bergshoeff, Y. Lozano and T. Ortin, *Massive branes*, *Nucl. Phys.* **B 518** (1998) 363 [hep-th/9712115].

[12] N. Kaloper, R.R. Khuri and R.C. Myers, *On generalized axion reductions*, *Phys. Lett.* **B 428** (1998) 297 [hep-th/9803066].

[13] E. Bergshoeff and J.P. van der Schaar, *On M9-branes*, *Class. and Quant. Grav.* **16** (1999) 23 [hep-th/9806069].

[14] P. Meessen and T. Ortin, *An* $\mathrm{SL}(2,\mathbb{Z})$ *multiplet of nine-dimensional type-II supergravity theories*, *Nucl. Phys.* **B 541** (1999) 195 [hep-th/9806120].

[15] C.M. Hull, *Massive string theories from M-theory and F-theory*, *J. High Energy Phys.* **11** (1998) 027 [hep-th/9811021].

[16] N. Kaloper and R.C. Myers, *The O(dd) story of massive supergravity*, *J. High Energy Phys.* **05** (1999) 010 [hep-th/9901045].

[17] M. Cvetič, H. Lü and C.N. Pope, *Gauged six-dimensional supergravity from massive type IIA*, *Phys. Rev. Lett.* **83** (1999) 5226 [hep-th/9906221]; *Brane-world Kaluza-Klein reductions and branes on the brane*, hep-th/0009183.

[18] A. Strominger, *Superstrings with torsion*, *Nucl. Phys.* **B 274** (1986) 253.

[19] B. de Wit, D.J. Smit and N.D. H. Dass, *Residual supersymmetry of compactified* $D = 10$ *supergravity*, *Nucl. Phys.* **B 283** (1987) 165.

[20] J. Polchinski and A. Strominger, *New vacua for type-II string theory*, *Phys. Lett.* **B 388** (1996) 736 [hep-th/9510227].

[21] E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070].

[22] K. Becker and M. Becker, *M-theory on eight-manifolds*, *Nucl. Phys.* **B 477** (1996) 155 [hep-th/9605053].

[23] J. Michelson, *Compactifications of type-IIB strings to four dimensions with non-trivial classical potential*, *Nucl. Phys.* **B 495** (1997) 127 [hep-th/9610151].

[24] A. Lukas, B.A. Ovrut and D. Waldram, *Stabilizing dilaton and moduli vacua in string and M-theory cosmology*, *Nucl. Phys.* **B 509** (1998) 169 [hep-th/9611204].

[25] H.P. Nilles, M. Olechowski and M. Yamaguchi, *Supersymmetry breaking and soft terms in M-theory*, *Phys. Lett.* **B 415** (1997) 24 [hep-th/9707143].

[26] A. Lukas, B.A. Ovrut and D. Waldram, *On the four-dimensional effective action of strongly coupled heterotic string theory*, *Nucl. Phys.* **B 532** (1998) 43 [hep-th/9710208].

JHEP04(2001)040

[27] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Heterotic M-theory in five dimensions*, *Nucl. Phys.* **B 552** (1999) 246 [hep-th/9806051].

[28] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070].

[29] K. Dasgupta, G. Rajesh and S. Sethi, *M-theory, orientifolds and G-flux*, *J. High Energy Phys.* **08** (1999) 023 [hep-th/9908088].

[30] S. Gukov, *Solitons, superpotentials and calibrations*, *Nucl. Phys.* **B 574** (2000) 169 [hep-th/9911011].

[31] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152].

[32] K. Behrndt and S. Gukov, *Domain walls and superpotentials from M-theory on Calabi-Yau three-folds*, *Nucl. Phys.* **B 580** (2000) 225 [hep-th/0001082].

[33] P. Mayr, *On supersymmetry breaking in string theory and its realization in brane worlds*, *Nucl. Phys.* **B 593** (2001) 99 [hep-th/0003198].

[34] M. Spalinski and T.R. Taylor, *Branes and fluxes in $D = 5$ Calabi-Yau compactifications of M-theory*, *Phys. Lett.* **B 485** (2000) 263 [hep-th/0004095].

[35] B.R. Greene, K. Schalm and G. Shiu, *Warped compactifications in M- and F-theory*, *Nucl. Phys.* **B 584** (2000) 480 [hep-th/0004103].

[36] M. Haack, J. Louis and M. Marquart, *Type-IIA and heterotic string vacua in $D = 2$*, hep-th/0011075.

[37] G. Curio and A. Krause, *Four-flux and warped heterotic M-theory compactifications*, hep-th/0012152.

[38] G. Curio, A. Klemm, D. Lüst and S. Theisen, *On the vacuum structure of type-II string compactifications on Calabi-Yau spaces with h-fluxes*, hep-th/0012213.

[39] J.A. Harvey and A. Strominger, *The heterotic string is a soliton*, *Nucl. Phys.* **B 449** (1995) 535 [hep-th/9504047].

[40] M.J. Duff, J.T. Liu and R. Minasian, *Eleven-dimensional origin of string/string duality: a one-loop test*, *Nucl. Phys.* **B 452** (1995) 261 [hep-th/9506126].

[41] A. Sen, *String string duality conjecture in six-dimensions and charged solitonic strings*, *Nucl. Phys.* **B 450** (1995) 103 [hep-th/9504027].

[42] D. Brecher and M.J. Perry, *Ricci-flat branes*, *Nucl. Phys.* **B 566** (2000) 151 [hep-th/9908018].

[43] E. Witten, *String theory dynamics in various dimensions*, *Nucl. Phys.* **B 443** (1995) 85 [hep-th/9503124].

[44] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions*, *Phys. Lett.* **B 76** (1978) 409.

[45] K. Bautier, S. Deser, M. Henneaux and D. Seminara, *No cosmological $D = 11$ supergravity*, *Phys. Lett.* **B 406** (1997) 49 [hep-th/9704131].

[46] S. Deser, *Uniqueness of $D = 11$ supergravity*, hep-th/9712064.

JHEP04(2001)040